MULTIAGENT SYSTEMS

Algorithmic, Game-Theoretic, and Logical Foundations

Yoav Shoham Stanford University

Kevin Leyton-Brown University of British Columbia

© all rights reserved

March 28, 2008

Credits and Acknowledgments xi

Introduction xiii

- 1 Distributed Constraint Satisfaction 1
 - 1.1 Defining distributed constraint satisfaction problems 2
 - 1.2 Domain-pruning algorithms 5
 - 1.3 Heuristic search algorithms 9
 - 1.3.1 The asynchronous backtracking algorithm 10
 - 1.3.2 A simple example 12
 - 1.3.3 An extended example: the four-queen problem 13
 - 1.3.4 Beyond the ABT algorithm
 - 1.4 History and references 18

2 Distributed Optimization 19

- 2.1 Distributed dynamic programming for path planning 19
 - 2.1.1 Asynchronous dynamic programming 19
 - 2.1.2 Learning real-time A* 21
- 2.2 Action selection in multiagent MDPs 22
- 2.3 Negotiation, auctions and optimization 28
 - 2.3.1 Introduction: from contract nets to auction-like optimization 28

17

- 2.3.2 The assignment problem and linear programming 31
- 2.3.3 The scheduling problem and integer programming 37
- 2.4 Social laws and conventions 45
- 2.5 History and references 47

3 Introduction to Noncooperative Game Theory: Games in Normal Form 49

- 3.1 Self-interested agents 49
 - 3.1.1 Example: friends and enemies 50
 - 3.1.2 Preferences and utility 51
- 3.2 Games in normal form 56
 - 3.2.1 Example: the TCP user's game 56

- 3.2.2 Definition of games in normal form 57
- 3.2.3 More examples of normal-form games 58
- 3.2.4 Strategies in normal-form games 61
- 3.3 Analyzing games: from optimality to equilibrium 63
 - 3.3.1 Pareto optimality 63
 - 3.3.2 Defining best response and Nash equilibrium 64

79

- 3.3.3 Finding Nash equilibria 65
- 3.3.4 Nash's theorem: proving the existence of Nash equilibria 68
- 3.4 Further solution concepts for normal-form games 75
 - 3.4.1 Maxmin and minmax strategies 76
 - 3.4.2 Minimax regret
 - 3.4.3 Removal of dominated strategies 81
 - 3.4.4 Rationalizability 84
 - 3.4.5 Correlated equilibrium 86
 - 3.4.6 Trembling-hand perfect equilibrium 88
 - 3.4.7 ϵ -Nash equilibrium 88
- 3.5 History and references 90
- 4 Computing Solution Concepts of Normal-Form Games 93
 - 4.1 Computing Nash equilibria of two-player, zero-sum games 93
 - 4.2 Computing Nash equilibria of two-player, general-sum games 95
 - 4.2.1 Complexity of computing a sample Nash equilibrium 96
 - 4.2.2 An LCP formulation and the Lemke–Howson algorithm 97
 - 4.2.3 Searching the space of supports 106
 - 4.2.4 Beyond sample equilibrium computation 109
 - 4.3 Computing Nash equilibria of *n*-player, general-sum games 110
 - 4.4 Computing maxmin and minmax strategies for two-player, general-sum games 113
 - 4.5 Identifying dominated strategies 114
 - 4.5.1 Domination by a pure strategy 115
 - 4.5.2 Domination by a mixed strategy 116
 - 4.5.3 Iterated dominance 118
 - 4.6 Computing correlated equilibria of *n*-player normal-form games 119
 - 4.7 History and references 121

5 Games with Sequential Actions: Reasoning and Computing with the Extensive Form 123

- 5.1 Perfect-information extensive-form games 123
 - 5.1.1 Definition 124
 - 5.1.2 Strategies and equilibria 125
 - 5.1.3 Subgame-perfect equilibrium 127
 - 5.1.4 Computing equilibria: backward induction 130
- 5.2 Imperfect-information extensive-form games 136
 - 5.2.1 Definition 136

© Shoham and Leyton-Brown, 2008

- 5.2.2 Strategies and equilibria 137
- 5.2.3 Computing equilibria: the sequence form 140
- 5.2.4 Sequential equilibrium 149
- 5.3 History and references 152
- 6 Richer Representations: Beyond the Normal and Extensive Forms 153
 - 6.1 Repeated games 154
 - 6.1.1 Finitely repeated games 155
 - 6.1.2 Infinitely repeated games 156
 - 6.1.3 "Bounded rationality": repeated games played by automata 159
 - 6.2 Stochastic games 166
 - 6.2.1 Definition 166
 - 6.2.2 Strategies and equilibria 167
 - 6.2.3 Computing equilibria 168
 - 6.3 Bayesian games 169
 - 6.3.1 Definition 171
 - 6.3.2 Strategies and equilibria 174
 - 6.3.3 Computing equilibria 177
 - 6.3.4 *Ex post* equilibria 180
 - 6.4 Congestion games 180
 - 6.4.1 Definition 181
 - 6.4.2 Computing equilibria 182
 - 6.4.3 Potential games 183
 - 6.4.4 Nonatomic congestion games 185
 - 6.4.5 Selfish routing and the price of anarchy 187
 - 6.5 Computationally motivated compact representations 192
 - 6.5.1 The expected utility problem 192
 - 6.5.2 Graphical games 195
 - 6.5.3 Action-graph games 197
 - 6.5.4 Multiagent influence diagrams 199
 - 6.5.5 GALA 202
 - 6.6 History and references 203
- 7 Learning and Teaching 205
 - 7.1 Why the subject of "learning" is complex 205
 - 7.1.1 The interaction between learning and teaching 205
 - 7.1.2 What constitutes learning? 207
 - 7.1.3 If learning is the answer, what is the question? 208
 - 7.2 Fictitious play 212
 - 7.3 Rational learning 217
 - 7.4 Reinforcement learning 222
 - 7.4.1 Learning in unknown MDPs 222
 - 7.4.2 Reinforcement learning in zero-sum stochastic games 223
 - 7.4.3 Beyond zero-sum stochastic games 226

Multiagent Systems, draft of March 28, 2008

	7.5	7.4.4 Belief-based reinforcement learning 227 No-regret learning and universal consistency 227
	76	Targeted learning 228
	77	Evolutionary learning and other large-population models 230
	/./	771 The replicator dynamic 231
		7.7.1 The replicator dynamic 251
		7.7.2 Evolutionality stable strategies 255
	7 0	1.1.5 Agent-based simulation and emergent conventions 257
	7.8	History and references 240
8	Com	nunication 243
	8.1	"Doing by talking" I: cheap talk 243
	8.2	"Talking by doing": signaling games 247
	8.3	"Doing by talking" II: speech-act theory 250
		8.3.1 Speech acts 250
		8.3.2 Rules of conversation 251
		8.3.3 A game-theoretic view of speech acts 253
		8.3.4 Applications 256
	8.4	History and references 260
•	4	Purformer Control Chaine 200
9	Aggre	gating Preferences: Social Choice 263
	9.1	Introduction 263
		9.1.1 Example: plurality voting 263
	9.2	A formal model 264
	9.3	Voting 266
		9.3.1 Voting methods 266
		9.3.2 Voting paradoxes 268
	9.4	Existence of social functions 270
		9.4.1 Social welfare functions 270
		9.4.2 Social choice functions 274
	9.5	Ranking systems 277
	9.6	History and references 281
10	Proto	cols for Strategic Agents: Mechanism Design 283
	10.1	Introduction 283
		10.1.1 Example: strategic voting 283
		10.1.2 Example: buying a shortest path 284
	10.2	Mechanism design with unrestricted preferences 285
		10.2.1 Implementation 286
		10.2.2 The revelation principle 288
		10.2.3 Impossibility of general dominant-strategy implementation 290
	10.3	Oussilinger preferences 290
	10.5	10.3.1 Rick attitudes 201
		10.3.2 Machanism design in the guardinear setting 204
	10.4	Efficient mechanisme 200
	10.4	Efficient mechanisms 270
		10.4.1 Groves mechanisms 299

 $\ensuremath{\textcircled{O}}$ Shoham and Leyton-Brown, 2008

- 10.4.2 The VCG mechanism 303 10.4.3 VCG and individual rationality 305 10.4.4 VCG and weak budget balance 307 10.4.5 Drawbacks of VCG 308 10.4.6 Budget balance and efficiency 312 10.4.7 The AGV mechanism 312 10.5 Beyond efficiency 313 10.5.1 What else can be implemented in dominant strategies? 10.5.2 Tractable Groves mechanisms 316 10.6 Computational applications of mechanism design 318 10.6.1 Task scheduling 318 321 10.6.2 Bandwidth allocation in computer networks 10.6.3 Multicast cost sharing 323 10.6.4 Two-sided matching 327 10.7 Constrained mechanism design 332 10.7.1 Contracts 333 10.7.2 Bribes 335 10.7.3 Mediators 336 10.8 History and references 337 11 Protocols for Multiagent Resource Allocation: Auctions 341 11.1 Single-good auctions 341 11.1.1 Canonical auction families 342 11.1.2 Auctions as Bayesian mechanisms 344 Second-price, Japanese, and English auctions 11.1.3 345 11.1.4 First-price and Dutch auctions 347 11.1.5 Revenue equivalence 350 11.1.6 **Risk** attitudes 352 11.1.7 Auction variations 354 11.1.8 "Optimal" (revenue-maximizing) auctions 356 11.1.9 Collusion 357 11.1.10 Interdependent values 361 11.2 Multiunit auctions 364 364 11.2.1 Canonical auction families 11.2.2 Single-unit demand 365 11.2.3 Beyond single-unit demand 368 370 11.2.4 Unlimited supply: random sampling auctions 11.2.5 Position auctions 372 11.3 Combinatorial auctions 375 11.3.1 Simple combinatorial auction mechanisms 377 11.3.2 The winner determination problem 378 11.3.3 Expressing a bid: bidding languages 381 11.3.4 Iterative mechanisms 386
 - 11.3.5 A tractable mechanism 389

Multiagent Systems, draft of March 28, 2008

314

- 11.4 Exchanges 390
 - 11.4.1 Two-sided auctions 391
 - 11.4.2 Prediction markets 392
- 11.5 History and references 393

12 Teams of Selfish Agents: An Introduction to Coalitional Game Theory 397

- 12.1 Coalitional games with transferable utility 397
 - 12.1.1 Definition 398
 - 12.1.2 Examples 398
 - 12.1.3 Classes of coalitional games 400
- 12.2 Analyzing coalitional games 401
 - 12.2.1 The Shapley value 402
 - 12.2.2 The core 405
 - 12.2.3 Refining the core: ϵ -core, least core, and nucleolus 408
- 12.3 Compact representations of coalitional games 411
 - 12.3.1 Weighted majority games and weighted voting games 411
 - 12.3.2 Weighted graph games 412
 - 12.3.3 Capturing synergies: a representation for superadditive games 414
 - 12.3.4 A decomposition approach: multi-issue representation 415
 - 12.3.5 A logical approach: marginal contribution nets 416
- 12.4 Further directions 418
 - 12.4.1 Alternative coalitional game models 418
 - 12.4.2 Advanced solution concepts 420
- 12.5 History and references 421

13 Logics of Knowledge and Belief 423

- 13.1 The partition model of knowledge 423
 - 13.1.1 Muddy children and warring generals 423
 - 13.1.2 Formalizing intuitions about the partition model 424
- 13.2 A detour to modal logic 427
 - 13.2.1 Syntax 428
 - 13.2.2 Semantics 429
 - 13.2.3 Axiomatics 429
 - 13.2.4 Modal logics with multiple modal operators 430
 - 13.2.5 Remarks about first-order modal logic 430
- 13.3 **S5**: An axiomatic theory of the partition model 431
- 13.4 Common knowledge, and an application to distributed systems 434
- 13.5 Doing time and an application to robotics 438
 - 13.5.1 Termination conditions for motion planning 438
 - 13.5.2 Coordinating robots 442
- 13.6 From knowledge to belief 444
- 13.7 Combining knowledge and belief (and revisiting knowledge) 445
- 13.8 History and references 451
- 14 Beyond Belief: Probability, Dynamics and Intention 453

© Shoham and Leyton-Brown, 2008

viii

- 14.1 Knowledge and probability 453
- 14.2 Dynamics of knowledge and belief 458
 - 14.2.1 Belief revision 458
 - 14.2.2 Beyond AGM: update, arbitration, fusion, and friends 464
 - 14.2.3 Theories of belief change: a summary 469
- 14.3 Logic, games, and coalition logic 470
- 14.4 Towards a logic of "intention" 471
 - 14.4.1 Some preformal intuitions 472
 - 14.4.2 The road to hell: elements of a formal theory of intention 474
 - 14.4.3 Group intentions 478
- 14.5 History and references 480

Appendices: Technical Background 481

A Probability Theory 483

- A.1 Probabilistic models 483
- A.2 Axioms of probability theory 483
- A.3 Marginal probabilities 484
- A.4 Conditional probabilities 484

B Linear and Integer Programming 485

- B.1 Linear programs 485
- B.2 Integer programs 487

C Markov Decision Problems (MDPs) 491

- C.1 The model 491
- C.2 Solving known MDPs via value iteration 491

D Classical Logic 493

- D.1 Propositional calculus 493
- D.2 First-order logic 494

Bibliography 497

Index 519